Magneto-optic polar Kerr and Faraday effects in periodic multilayers

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Abstract: Magnetooptic (MO) effects in magnetic multilayers with periodically stratified regions are analyzed for the case of normal light wave incidence and polar magnetization (Faraday and polar Kerr effects). From the universal 4×4-matrix formalism simplified analytical representations restricted to terms linear in the off-diagonal permittivity tensor elements are obtained with no loss in accuracy. The MO effects are expressed as weighted sums of contributions from individual layers. Approximate expressions useful for the evaluation of trends in MO effects are given for periodic multilayers consisting of blocks with ultrathin magnetic films. The procedure is illustrated on periodic systems built of symmetric units. Limits on the ultrathin approximation are discussed.

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1 Introduction

Magnetooptical (MO) technique is an important tool in diagnostics of structures containing magnetic ultrathin films [1]. Thanks to an advanced technology these systems display a precisely defined composition profile and their magneto-optical response can be often adequately described by electromagnetic wave theory [2]. The most complete information is provided by the universal 4×4 matrix approach [3, 4, 5] which allows computer simulations without any restriction on the permittivity tensor in the individual layers as well as on the angle of incidence of the optical wave and the number of layers in the system.

The purpose of the present paper is to provide simplified analytical representations for the MO response in magnetic multilayers, which display some periodicity in their profile. We consider the case of normal light incidence and polar magnetization (Faraday and polar Kerr effects). The analysis makes use of previous results for periodically stratified structures [6, 7, 8]. Originally, the approach has been developed for isotropic multilayers, where the eigen modes are linearly polarized (LP) perpendicular (s) and parallel (p) to the plane of incidence. It can be extended to periodic multilayers with arbitrary modes provided the mode conversion is absent. This covers a number of cases in optically anisotropic layered media.

Here we are concerned with the isotropic media subjected to a uniform magnetization leading to circularly polarized (CP) eigen modes when the wave, characterized by the wavevector, $\vec{\gamma}$, propagates parallel to the magnetization vector, $\vec{M}$. The approach can be also applied to the MO cases with $\vec{M}$ perpendicular to $\vec{\gamma}$, leading to LP eigen modes. In most cases, the off-diagonal element of permittivity tensor in the n-the layer, $\varepsilon_{xy}^{(n)}$, is much smaller than the diagonal one, $\varepsilon_{xx}^{(n)}$. Thanks to a justified restriction to the terms linear in $\varepsilon_{xy}^{(n)}$, the MO polar Kerr and Faraday effects in multilayers can be split into a sum of contributions from individual layers [9]. This is helpful in the analysis of observed effects and may be applied, *e.g.*, for the study of in-depth magnetization profiles [10]. Approximate expressions can be derived for multilayers consisting of ultrathin magnetic layers [11, 12].

Definitions of the basic MO quantities are summarized in Sec. 2. The extension of the theory to periodic magnetic multilayers and the analytic formulae for the corresponding MO effects are presented in Sec. 3. The procedures are illustrated on multilayers built of symmetric units. Examples of modeling for the MO response in periodic systems containing ultrathin cobalt magnetic films are given in Sec. 4.

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2 Polar magneto-optics in multilayers

2.1 4×4 *Matrix approach*

We first summarize the 4×4 matrix formalism [3, 4] applied here to the description of the MO interactions at normal light incidence in a stack of layers subjected to magnetization perpendicular to the planar interfaces (polar configuration). We consider the structure consisting of $\mathcal{N}$ homogeneous layers separated by interface planes $z = z_n$ $(n = 1, ..., \mathcal{N})$ in a Cartesian coordinate system, $z_{n-1} < z_n$. The structure is sandwiched between semi–infinite media $z < z_0$ and $z > z_{\mathcal{N}}$ indexed 0 and $\mathcal{N} + 1$, respectively. The n-th layer confined by the interface planes $z = z_{n-1}$ and $z = z_n$ is characterized by relative permittivity tensor of an originally isotropic medium uniformly magnetized perpendicular to the interface planes $z =$ const., *i.e.*, along the z-axis (polar configuration)

$$\overleftrightarrow{\varepsilon}^{(n)} = \begin{pmatrix} \varepsilon_{xx}^{(n)} & \varepsilon_{xy}^{(n)} & 0 \\ -\varepsilon_{xy}^{(n)} & \varepsilon_{xx}^{(n)} & 0 \\ 0 & 0 & \varepsilon_{zz}^{(n)} \end{pmatrix}. \tag{1}$$

The magnetic permeability in the layers is assumed to take its vacuum value. For plane wave which propagates parallel to the z-axis in the n-th layer medium, the wave equation provides

$$\begin{bmatrix} \left(\frac{c}{\omega}\gamma_z^{(n)}\right)^2 - \varepsilon_{xx}^{(n)} & -\varepsilon_{xy}^{(n)} & 0 \\ \varepsilon_{xy}^{(n)} & \left(\frac{c}{\omega}\gamma_z^{(n)}\right)^2 - \varepsilon_{xx}^{(n)} & 0 \\ 0 & 0 & \varepsilon_{zz}^{(n)} \end{bmatrix} \begin{bmatrix} E_{0x}^{(n)} \\ E_{0y}^{(n)} \\ E_{0z}^{(n)} \end{bmatrix} = 0, \tag{2}$$

where $E_{0i}^{(n)}$, $i = x, y$ and z are Cartesian components of complex amplitude vector $\boldsymbol{E}_0^{(n)}$ and $\vec{\gamma}^{(n)} = \left(0, \quad 0, \quad \gamma_z^{(n)}\right)$ is the complex wavevector, $\vec{\gamma}^{(n)} = \left(\frac{\omega}{c}\right) N^{(n)}\hat{\mathbf{z}}$ where $N^{(n)}$ is the complex index of refraction in magnetic medium.

According to Eq. (2), the four eigen values of $\gamma_{zj}^{(n)}, j = 1, ..., 4$ and the associated normalized and orthogonal electric field CP eigen modes $\hat{\mathbf{e}}_j^{(n)}$ (as determined in a coordinate frame fixed to the laboratory including the sense of layer magnetization specified by $\varepsilon_{xy}^{(n)}$) are $\gamma_{z\,1,2}^{(n)} = \pm\frac{\omega}{c}N_+^{(n)}$, $\gamma_{z\,3,4}^{(n)} = \pm\frac{\omega}{c}N_-^{(n)}$, $\hat{\mathbf{e}}_1^{(n)} = \hat{\mathbf{e}}_2^{(n)} = (\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$, $\hat{\mathbf{e}}_3^{(n)} = \hat{\mathbf{e}}_4^{(n)} = (\hat{\mathbf{x}} - \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$, where $\left(N_\pm^{(n)}\right)^2 = \varepsilon_{xx}^{(n)} \pm \mathrm{i}\varepsilon_{xy}^{(n)}$, and $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are Cartesian unit vectors. The CP field amplitudes in the semi–infinite media sandwiching the layered structures at $z = z_0$ and $z = z_{\mathcal{N}}$ are related by

$$\mathbf{E}_0^{(0)} = \begin{bmatrix} \mathrm{E}_{01}^{(0)} \\ \mathrm{E}_{02}^{(0)} \\ \mathrm{E}_{03}^{(0)} \\ \mathrm{E}_{04}^{(0)} \end{bmatrix} = \begin{bmatrix} \mathrm{M}_{11} & \mathrm{M}_{12} & 0 & 0 \\ \mathrm{M}_{21} & \mathrm{M}_{22} & 0 & 0 \\ 0 & 0 & \mathrm{M}_{33} & \mathrm{M}_{34} \\ 0 & 0 & \mathrm{M}_{43} & \mathrm{M}_{44} \end{bmatrix} \begin{bmatrix} \mathrm{E}_{01}^{(\mathcal{N}+1)} \\ \mathrm{E}_{02}^{(\mathcal{N}+1)} \\ \mathrm{E}_{03}^{(\mathcal{N}+1)} \\ \mathrm{E}_{04}^{(\mathcal{N}+1)} \end{bmatrix} = \mathbf{M}\mathbf{E}_0^{(\mathcal{N}+1)}. \tag{3}$$

Here $\mathrm{E}_{01}^{(0)}$ and $\mathrm{E}_{03}^{(0)}$ represent the complex amplitudes determined at the interface $z = z_0$ in the semi–infinite medium 0 (specified by $z < z_0$) for eigen modes propagating in the positive sense of the z-axis with polarizations $(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ and $(\hat{\mathbf{x}} - \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$. The amplitudes $\mathrm{E}_{02}^{(0)}$ and $\mathrm{E}_{04}^{(0)}$ at the same interface belong to eigen modes with $(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ and $(\hat{\mathbf{x}} - \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ but propagating in the negative sense of the z-axis. In the semi–infinite medium $\mathcal{N} + 1$ (specified by $z > z_{\mathcal{N}}$) $\mathrm{E}_{01}^{(\mathcal{N}+1)}$ and $\mathrm{E}_{03}^{(\mathcal{N}+1)}$ denote respectively the amplitudes at the interface $z = z_{\mathcal{N}}$ of the eigen modes with $(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ and $(\hat{\mathbf{x}} - \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ propagating in the positive sense of the z-axis. The amplitudes $\mathrm{E}_{02}^{(\mathcal{N}+1)}$

and $E_{04}^{(\mathcal{N}+1)}$ at the same interface correspond respectively to the eigen modes with $(\hat{\mathbf{x}} + \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ and $(\hat{\mathbf{x}} - \mathrm{i}\hat{\mathbf{y}})/\sqrt{2}$ propagating in the negative sense of the z-axis. We assume $E_{02}^{(\mathcal{N}+1)} = E_{04}^{(\mathcal{N}+1)} = 0$. Then $E_{01}^{(0)}$ and $E_{03}^{(0)}$ are the mode amplitudes of a single pair of CP waves incident (i) on the structure and we denote $E_{01}^{(0)} = E_{+}^{(i)}, \quad E_{03}^{(0)} = E_{-}^{(i)}$. There are also single pairs of CP transmitted (t) and reflected (r) waves labelled respectively $E_{01}^{(\mathcal{N}+1)} = E_{+}^{(t)}, \quad E_{03}^{(\mathcal{N}+1)} = E_{-}^{(t)}$ and $E_{02}^{(0)} = E_{+}^{(r)}, \quad E_{04}^{(0)} = E_{-}^{(r)}$.

The matrix $\mathbf{M}$ representing the structure is given by the product

$$\mathbf{M} = \left[\mathbf{D}^{(0)}\right]^{-1} \prod_{n=1}^{\mathcal{N}} \mathbf{S}^{(n)} \mathbf{D}^{(\mathcal{N}+1)}, \tag{4}$$

where the block diagonal medium matrix $\mathbf{S}^{(n)}$ is given by

$$\begin{aligned} \mathbf{S}^{(n)} &= \mathbf{D}^{(n)} \mathbf{P}^{(n)} \left[\mathbf{D}^{(n)}\right]^{-1} = \qquad (5) \\ &= \begin{bmatrix} \cos\beta_{+}^{(n)} & \mathrm{i}N_{+}^{(n)-1}\sin\beta_{+}^{(n)} & 0 & 0 \\ \mathrm{i}N_{+}^{(n)}\sin\beta_{+}^{(n)} & \cos\beta_{+}^{(n)} & 0 & 0 \\ 0 & 0 & \cos\beta_{-}^{(n)} & \mathrm{i}N_{-}^{(n)-1}\sin\beta_{-}^{(n)} \\ 0 & 0 & \mathrm{i}N_{-}^{(n)}\sin\beta_{-}^{(n)} & \cos\beta_{-}^{(n)} \end{bmatrix} \end{aligned}$$

for $n = 1, 2, \cdots$, and $\mathcal{N}$. Here

$$\beta_{\pm}^{(n)} = \frac{\omega}{c} N_{\pm}^{(n)} d_n, \tag{6}$$

d_n denotes the thickness of n-th layer. For a non-magnetic layer $N_{+}^{(n)} = N_{-}^{(n)}$. The dynamic and propagation matrices are defined respectively as

$$\mathbf{D}^{(n)} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ N_{+}^{(n)} & -N_{+}^{(n)} & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & N_{-}^{(n)} & -N_{-}^{(n)} \end{bmatrix} \tag{7}$$

for $n = 1, 2, \cdots, \mathcal{N}$, and $\mathcal{N}+1$, and

$$\mathbf{P}^{(n)} = \begin{bmatrix} \exp\left(\mathrm{i}\beta_{+}^{(n)}\right) & 0 & 0 & 0 \\ 0 & \exp\left(-\mathrm{i}\beta_{+}^{(n)}\right) & 0 & 0 \\ 0 & 0 & \exp\left(\mathrm{i}\beta_{-}^{(n)}\right) & 0 \\ 0 & 0 & 0 & \exp\left(-\mathrm{i}\beta_{-}^{(n)}\right) \end{bmatrix}, \tag{8}$$

for $n = 1, 2, \cdots$, and $\mathcal{N}$.

2.2 *Jones transmission and reflection matrices*

The $\mathbf{M}$-matrix allows one to compute the amplitude reflection and transmission coefficients as well as to determine the changes in the polarization state of the incident wave. Circularly polarized transmission and reflection 2×2 Jones matrices are defined as [13]

$$\begin{bmatrix} E_{+}^{(t)} \\ E_{-}^{(t)} \end{bmatrix} = \begin{bmatrix} t_{+} & 0 \\ 0 & t_{-} \end{bmatrix} \begin{bmatrix} E_{+}^{(i)} \\ E_{-}^{(i)} \end{bmatrix}, \tag{9}$$

$$\begin{bmatrix} E_+^{(r)} \\ E_-^{(r)} \end{bmatrix} = \begin{bmatrix} r_+ & 0 \\ 0 & r_- \end{bmatrix} \begin{bmatrix} E_+^{(i)} \\ E_-^{(i)} \end{bmatrix}, \tag{10}$$

where $t_\pm$ and $r_\pm$ are the CP transmission and reflection coefficients. For a multilayer structure represented by **M**-matrix they can be obtained from $t_+ = (M_{11})^{-1}$, $t_- = (M_{33})^{-1}$, $r_+ = M_{21}/M_{11}$, $r_- = M_{43}/M_{33}$. The Cartesian transmission and reflection matrices take the form

$$[t] = \begin{bmatrix} t_{xx} & t_{xy} \\ -t_{xy} & t_{xx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (t_- + t_+) & \mathrm{i}\,(t_- - t_+) \\ -\mathrm{i}\,(t_- - t_+) & (t_- + t_+) \end{bmatrix}, \tag{11}$$

$$[r] = \begin{bmatrix} r_{xx} & r_{xy} \\ -r_{xy} & r_{xx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (r_- + r_+) & \mathrm{i}\,(r_- - r_+) \\ -\mathrm{i}\,(r_- - r_+) & (r_- + r_+) \end{bmatrix}. \tag{12}$$

We have the following relations between the Cartesian Jones vectors of the incident, transmitted and reflected waves

$$\begin{bmatrix} E_x^{(t)} \\ E_y^{(t)} \end{bmatrix} = [t] \begin{bmatrix} E_x^{(i)} \\ E_y^{(i)} \end{bmatrix}, \tag{13}$$

$$\begin{bmatrix} E_x^{(r)} \\ E_y^{(r)} \end{bmatrix} = [r] \begin{bmatrix} E_x^{(i)} \\ E_y^{(i)} \end{bmatrix} \tag{14}$$

determined from the experiment using, *e.g.*, the polarization modulation technique [13, 14, 15]. For an incident wave linearly polarized parallel to the x-axis ($\chi_i = 0$), these relations characterize the complex Faraday effect, and the complex polar Kerr effect, respectively, which can be expressed as

$$\chi_t = -\frac{t_{xy}}{t_{xx}} = \mathrm{i}\frac{t_+ - t_-}{t_+ + t_-} = -\mathrm{i}\frac{M_{11} - M_{33}}{M_{11} + M_{33}}, \tag{15}$$

$$\chi_r = -\frac{r_{xy}}{r_{xx}} = \mathrm{i}\frac{r_+ - r_-}{r_+ + r_-} = -\mathrm{i}\frac{M_{43}M_{11} - M_{21}M_{33}}{M_{43}M_{11} + M_{21}M_{33}}. \tag{16}$$

2.3 *Simplified expressions for MO effects*

Based on the **M**–matrix, we can obtain simplified analytical representations making use of the restrictions to:

(**a**) to MO effects linear in the off-diagonal permittivity tensor elements justified when $\varepsilon_{xy}^{(n)} \ll \varepsilon_{xx}^{(n)}$,

(**b**) to small MO azimuth rotations, θ_l, and ellipticities, ϵ_l, then $\chi_l \approx \theta_l + \mathrm{i}\epsilon_l$, $(l = t, r)$,

(**c**) to a layer thickness much smaller than the radiation wavelength (in the n-th layer medium) which sometimes justifies the development $\exp\left(-2\mathrm{i}\frac{\omega}{c}N^{(n)}d_n\right) \approx 1 - 2\mathrm{i}\frac{\omega}{c}N^{(n)}d_n$ (ultrathin film approximation).

We start from the expressions for the off-diagonal elements of the transmission and reflection matrices in Cartesian representation given in terms of the corresponding matrix elements in circular representation according to Eqs. (11) and (12) as

$$\begin{aligned} t_{xy}^{(0,\mathcal{N}+1)} &= -\frac{\mathrm{i}}{2}\left(t_+^{(0,\mathcal{N}+1)} - t_-^{(0,\mathcal{N}+1)}\right), \\ r_{xy}^{(0,\mathcal{N}+1)} &= -\frac{\mathrm{i}}{2}\left(r_+^{(0,\mathcal{N}+1)} - r_-^{(0,\mathcal{N}+1)}\right). \end{aligned} \tag{17}$$

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They are odd functions in the off-diagonal tensor elements. We define

$$\Delta N^{(n)} = \frac{1}{2}\left(N_+^{(n)} - N_-^{(n)}\right) = \frac{i\varepsilon_{xy}^{(n)}}{2N^{(n)}}, \tag{18}$$

where $N^{(n)} \approx (N_+^{(n)} + N_-^{(n)})/2$. The diagonal elements are even in $\varepsilon_{xy}^{(n)}$

$$\begin{aligned} t_{xx}^{(0,\mathcal{N}+1)} &= \frac{1}{2}\left(t_+^{(0,\mathcal{N}+1)} + t_-^{(0,\mathcal{N}+1)}\right), \\ r_{xx}^{(0,\mathcal{N}+1)} &= \frac{1}{2}\left(r_+^{(0,\mathcal{N}+1)} + r_-^{(0,\mathcal{N}+1)}\right), \end{aligned} \tag{19}$$

and they are not sensitive, to the first order in $\varepsilon_{xy}^{(n)}$, to magnetic ordering.

To first order in $\Delta N^{(n)}$ the elements of the Cartesian transmission and reflection matrices given in Eq. (17) can be obtained by differentiation [9]

$$\begin{aligned} t_{xy}^{(0,\mathcal{N}+1)} &= -\mathrm{i}\Delta t_{xx}^{(0,\mathcal{N}+1)} = -\mathrm{i}\sum_{n=1}^{\mathcal{N}} \frac{\partial t_{xx}^{(0,\mathcal{N}+1)}}{\partial N_n}\Delta N_n, \\ r_{xy}^{(0,\mathcal{N}+1)} &= -\mathrm{i}\Delta r_{xx}^{(0,\mathcal{N}+1)} = -\mathrm{i}\sum_{n=1}^{\mathcal{N}} \frac{\partial r_{xx}^{(0,\mathcal{N}+1)}}{\partial N_n}\Delta N_n, \end{aligned} \tag{20}$$

where we have replaced $N^{(n)}$ and $\Delta N^{(n)}$ by N_n and ΔN_n, respectively. The off-diagonal transmission or reflection matrix elements are expressed as a sum of contributions from individual layers, each proportional to ΔN_n (in non-magnetic layers $\Delta N_n = 0$). According to Eqs. (20) they can be obtained from the Cartesian transmission and reflection coefficients $t_{xx}^{(0,\mathcal{N}+1)}$ and $r_{xx}^{(0,\mathcal{N}+1)}$. The Cartesian transmission and reflection coefficients can be obtained from the **M**-matrix of the structure corresponding to the isotropic case

$$\begin{aligned} t_{xx}^{(0,\mathcal{N}+1)} &= \frac{1}{\mathrm{M}_{11}}, \qquad t_{xy}^{(0,\mathcal{N}+1)} = \mathrm{i}\frac{\Delta\left(\mathrm{M}_{11}\right)}{\mathrm{M}_{11}^2} = \frac{\mathrm{i}}{\mathrm{M}_{11}^2}\sum_{n=1}^{\mathcal{N}}\frac{\partial \mathrm{M}_{11}}{\partial N_n}\Delta N_n, \\ r_{xx}^{(0,\mathcal{N}+1)} &= \frac{\mathrm{M}_{21}}{\mathrm{M}_{11}}, \\ r_{xy}^{(0,\mathcal{N}+1)} &= -\mathrm{i}\Delta\left(\frac{\mathrm{M}_{21}}{\mathrm{M}_{11}}\right) = \frac{\mathrm{i}}{\mathrm{M}_{11}^2}\left(\mathrm{M}_{21}\sum_{n=1}^{\mathcal{N}+1}\frac{\partial \mathrm{M}_{11}}{\partial N_n} - \mathrm{M}_{11}\sum_{n=1}^{\mathcal{N}+1}\frac{\partial \mathrm{M}_{21}}{\partial N_n}\right)\Delta N_n \end{aligned} \tag{21}$$

where $N_+^{(n)} = N_-^{(n)}$ for $n = 0, \cdots, \mathcal{N}+1$, and $\mathrm{M}_{11} = \mathrm{M}_{33}$, $\mathrm{M}_{21} = \mathrm{M}_{43}$.

For $\chi_i = 0$, the complex number representation of the polarization state of the transmitted and reflected waves can be expressed according to Eqs. (15,16) as

$$\chi_t^{(0,\mathcal{N}+1)} = -\frac{t_{xy}}{t_{xx}} = -\mathrm{i}\frac{\Delta\left(\mathrm{M}_{11}\right)}{\mathrm{M}_{11}} = -\mathrm{i}\sum_{n=1}^{\mathcal{N}+1}\frac{\partial \ln\left(\mathrm{M}_{11}\right)}{\partial N_n}\Delta N_n \tag{22}$$

and

$$\chi_r^{(0,\mathcal{N}+1)} = -\frac{r_{xy}}{r_{xx}} = \mathrm{i}\frac{\mathrm{M}_{11}\Delta\left(\mathrm{M}_{21}\right) - \mathrm{M}_{21}\Delta\left(\mathrm{M}_{11}\right)}{\mathrm{M}_{11}\mathrm{M}_{21}} = \mathrm{i}\sum_{n=1}^{\mathcal{N}+1}\frac{\partial}{\partial N_n}\left[\ln\left(\frac{\mathrm{M}_{21}}{\mathrm{M}_{11}}\right)\right]\Delta N_n. \tag{23}$$

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3 Multilayers with periodic regions

The representation of $\mathbf{M}$ in terms of a product of $\mathbf{S}^{(n)}$ in Eq. (4) is suitable for the treatment of multilayers containing a periodically stratified region. To this purpose we write

$$\mathbf{M} = \mathbf{C}\mathbf{L}^q\mathbf{W}, \tag{24}$$

where $\mathbf{C}$ represents the incident medium and cover layer(s), $\mathbf{L}^q$ is the periodic region with the unit $\mathbf{L}$ repeated q times and $\mathbf{W}$ the buffer layer(s), substrate and the exit medium. The $\mathbf{L}$–matrix itself can be considered as a product of matrices representing homogenous layers and can be expressed as

$$\mathbf{L} = \begin{bmatrix} m_{11}^{+} & m_{12}^{+} & 0 & 0 \\ m_{21}^{+} & m_{22}^{+} & 0 & 0 \\ 0 & 0 & m_{11}^{-} & m_{12}^{-} \\ 0 & 0 & m_{21}^{-} & m_{22}^{-} \end{bmatrix}. \tag{25}$$

With restriction to symmetric units, where $m_{11}^{\pm} = m_{22}^{\pm}$, the q-th power of $\mathbf{L}$ is then given by [6, 7, 8]

$$\mathbf{L}^q = \begin{bmatrix} m_{11}^{+}p_q^{+} - p_{q-1}^{+} & m_{12}^{+}p_q^{+} & 0 & 0 \\ m_{21}^{+}p_q^{+} & m_{11}^{+}p_q^{+} - p_{q-1}^{+} & 0 & 0 \\ 0 & 0 & m_{11}^{-}p_q^{-} - p_{q-1}^{-} & m_{12}^{-}p_q^{-} \\ 0 & 0 & m_{21}^{-}p_q^{-} & m_{11}^{-}p_q^{-} - p_{q-1}^{-} \end{bmatrix}. \tag{26}$$

Here $p_q^{\pm}(m_{11}^{\pm})$ denote the Chebyshev polynomials of the second kind with the argument $m_{11}^{\pm}$ (see Appendix). In the simplest case of a multilayer built of symmetric trilayer units $A/B/A$ (Figure 1).

$$\mathbf{L} = \mathbf{S}^{(A)}\mathbf{S}^{(B)}\mathbf{S}^{(A)}, \tag{27}$$

where $\mathbf{S}^{(A)}$ and $\mathbf{S}^{(B)}$ are given by Eq. (5) with $n = A$ and $\beta_{\pm}^{(n)} = \beta_{\pm}^{(A)}/2$ for $\mathbf{S}^{(A)}$ and $\beta_{\pm}^{(n)} = \beta_{\pm}^{(B)}$ for $\mathbf{S}^{(B)}$. Then according to Eq. (6), d_A denotes the total thickness of the two identical sandwiching layers of medium A. The matrix elements of the symmetric unit $\mathbf{L}$ in Eq. (25) become

air	N_0		
A – nonmagnetic film	N_A	$d_A/2$	symmetric unit
B – magnetic film	N_B, ΔN_B	d_B	
A – nonmagnetic film	N_A	$d_A/2$	
$\vdots$			
A – nonmagnetic substrate	N_2		

Fig. 1. Symmetric $A/B/A$ unit.

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\begin{aligned}
m_{11}^{\pm} &= \cos\beta_{\pm}^{(A)}\cos\beta_{\pm}^{(B)} - \frac{1}{2}\left(\frac{N_{\pm}^{(A)}}{N_{\pm}^{(B)}} + \frac{N_{\pm}^{(B)}}{N_{\pm}^{(A)}}\right)\sin\beta_{\pm}^{(A)}\sin\beta_{\pm}^{(B)}, \\
m_{12}^{\pm} &= \frac{\mathrm{i}}{\mathrm{N}_{\pm}^{(\mathrm{A})}}\left[\sin\beta_{\pm}^{(A)}\cos\beta_{\pm}^{(B)} + \frac{1}{2}\left(\frac{N_{\pm}^{(A)}}{N_{\pm}^{(B)}} + \frac{N_{\pm}^{(B)}}{N_{\pm}^{(A)}}\right)\cos\beta_{\pm}^{(A)}\sin\beta_{\pm}^{(B)}\right. \\
&+ \left.\frac{1}{2}\left(\frac{N_{\pm}^{(A)}}{N_{\pm}^{(B)}} - \frac{N_{\pm}^{(B)}}{N_{\pm}^{(A)}}\right)\sin\beta_{\pm}^{(B)}\right], \\
m_{21}^{\pm} &= \mathrm{i}\mathrm{N}_{\pm}^{(\mathrm{A})}\left[\sin\beta_{\pm}^{(\mathrm{A})}\cos\beta_{\pm}^{(\mathrm{B})} + \frac{1}{2}\left(\frac{\mathrm{N}_{\pm}^{(\mathrm{A})}}{\mathrm{N}_{\pm}^{(\mathrm{B})}} + \frac{\mathrm{N}_{\pm}^{(\mathrm{B})}}{\mathrm{N}_{\pm}^{(\mathrm{A})}}\right)\cos\beta_{\pm}^{(\mathrm{A})}\sin\beta_{\pm}^{(\mathrm{B})}\right. \\
&- \left.\frac{1}{2}\left(\frac{N_{\pm}^{(A)}}{N_{\pm}^{(B)}} - \frac{N_{\pm}^{(B)}}{N_{\pm}^{(A)}}\right)\sin\beta_{\pm}^{(B)}\right]. \qquad (28)
\end{aligned}
$$

3.1 *Analytical representations*

In order to obtain the MO characteristics of the magnetic multilayer with periodic regions it is sufficient to apply Eqs. (21), (22) and (23) to its 2×2 isotropic equivalent representation. For the purpose of an illustration we consider the periodic structure with a symmetric unit represented by $\mathbf{L}$ repeated q times and sandwiched between semi-infinite isotropic media 0 and 2. The extension to more complicated sandwiching structures (including cover, buffer, and substrate layers) can be done by the insertion of the corresponding matrices to the product. In order to find $\chi_t^{(0,qL,2)}$ and $\chi_r^{(0,qL,2)}$ we start from Eq. (24) for $N_+^{(n)} = N_-^{(n)}$ and consider 2×2 matrix product representing the multilayer in the absence of magnetic order

$$
\mathbf{M} = \mathbf{C}\mathbf{L}^q\mathbf{W} = \frac{1}{2N_0}\begin{pmatrix} N_0 & 1 \\ N_0 & -1 \end{pmatrix}\begin{pmatrix} \mathrm{L}_{11}^{(q)} & \mathrm{L}_{12}^{(q)} \\ \mathrm{L}_{21}^{(q)} & \mathrm{L}_{11}^{(q)} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ N_2 & -N_2 \end{pmatrix}. \qquad (29)
$$

where, according to Eq. (26) $L_{11}^{(q)} = m_{11}p_q - p_{q-1}$, $L_{12}^{(q)} = m_{12}p_q$, $L_{21}^{(q)} = m_{21}p_q$.
With help of Eqs. (22,23) the MO Faraday and Kerr effects of this periodic structure are given by

$$
\begin{aligned}
\chi_t^{(0,qL,2)} &= -\mathrm{i}\left\{p_q\left[N_0\left(m_{11} + N_2 m_{12}\right) + \left(N_2 m_{11} + m_{21}\right)\right] - p_{q-1}\left(N_0 + N_2\right)\right\}^{-1} \\
&\times\Big\{\left[\left(p_q - p'_{q-1}\right)\left(N_0 + N_2\right) + p'_q N_0\left(m_{11} + N_2 m_{12}\right)\right. \\
&\left.+ p'_q\left(N_2 m_{11} + m_{21}\right)\right]\Delta m_{11} + p_q\left(N_0 N_2 \Delta m_{12} + \Delta m_{21}\right)\Big\}, \qquad (30) \\
\chi_r^{(0,qL,2)} &= -2\mathrm{i}N_0\left\{p_q\left[N_0\left(m_{11} + N_2 m_{12}\right) + \left(N_2 m_{11} + m_{21}\right)\right] - p_{q-1}\left(N_0 + N_2\right)\right\}^{-1} \\
&\times\left\{p_q\left[N_0\left(m_{11} + N_2 m_{12}\right) - \left(N_2 m_{11} + m_{21}\right)\right] - p_{q-1}\left(N_0 - N_2\right)\right\}^{-1} \\
&\times\Big\{\Delta m_{11}(N_2^2 m_{12} - m_{21})(p_q^2 + p'_q p_{q-1} - p_q p'_{q-1}) - \Delta m_{12} N_2 p_q \\
&\times\left[\left(N_2 m_{11} + m_{21}\right)p_q - N_2 p_{q-1}\right] + \Delta m_{21} p_q\left[p_q\left(m_{11} + N_2 m_{12}\right) - p_{q-1}\right]\Big\}. \qquad (31)
\end{aligned}
$$

The prime indicates the differentiation of the Chebyshev polynomials with respect to their argument m_{11}. The MO response is expressed as a sum of contributions from individual layers. This representation leads to the results which are equivalent to those computed with the 4×4 matrix formalism. For a simple symmetric unit $A/B/A$, characterized by the refractive indices N_A and N_B, the $\mathbf{L}$-matrix elements are given by

Eqs. (28) (with $m_{11}^{\pm}$, $m_{12}^{\pm}$, and $m_{21}^{\pm}$ replaced by m_{11}, m_{12}, and m_{21}, respectively),

$$\begin{aligned}
m_{11} &= U_{AB}\left[1 - r_{AB}^2 \mathrm{e}^{-2\mathrm{i}\beta_A} + \mathrm{e}^{-2\mathrm{i}\beta_B}\left(\mathrm{e}^{-2\mathrm{i}\beta_A} - r_{AB}^2\right)\right], & (32)\\
m_{12} &= \frac{U_{AB}}{N_A}\left[\left(1 + r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)^2 - \mathrm{e}^{-2\mathrm{i}\beta_B}\left(r_{AB} + \mathrm{e}^{-\mathrm{i}\beta_A}\right)^2\right], & (33)\\
m_{21} &= U_{AB}N_A\left[\left(1 - r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)^2 - \mathrm{e}^{-2\mathrm{i}\beta_B}\left(r_{AB} - \mathrm{e}^{-\mathrm{i}\beta_A}\right)^2\right]. & (34)
\end{aligned}$$

where $U_{AB} = \frac{1}{2} t_{AB}^{-1} t_{BA}^{-1} \mathrm{e}^{\mathrm{i}\beta_A}\mathrm{e}^{\mathrm{i}\beta_B}$. Here the interface Fresnel coefficients are given by

$$r_{n-1,n} = \frac{N_{n-1} - N_n}{N_{n-1} + N_n}, \quad \text{and} \quad t_{n-1,n} = \frac{2N_{n-1}}{N_{n-1} + N_n}. \tag{35}$$

We now assume that in the symmetric unit only the central layer B is magnetic and displays MO activity characterized by $\Delta N_B \neq 0$. Then $\Delta m_{ij} = (dm_{ij}/dN_B)\Delta N_B$. Using Eqs. (28) we obtain

$$\begin{aligned}
\Delta m_{11} &= \mathrm{i}\frac{\Delta N_B}{N_B} U_{AB}\left\{\beta_B\left[1 - r_{AB}^2\mathrm{e}^{-2\mathrm{i}\beta_A} - \mathrm{e}^{-2\mathrm{i}\beta_B}\left(\mathrm{e}^{-2\mathrm{i}\beta_A} - r_{AB}^2\right)\right]\right.\\
&+ \left.\mathrm{i} r_{AB}\left(1 - \mathrm{e}^{-2\mathrm{i}\beta_A}\right)\left(1 - \mathrm{e}^{-2\mathrm{i}\beta_B}\right)\right\}, & (36)\\
\Delta m_{12} &= \mathrm{i}\frac{\Delta N_B}{N_B} U_{AB}\frac{1}{N_A}\left\{\beta_B\left[\left(1 + r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)^2 + \mathrm{e}^{-2\mathrm{i}\beta_B}\left(r_{AB} + \mathrm{e}^{-\mathrm{i}\beta_A}\right)^2\right]\right.\\
&+ \left.\mathrm{i}\left(1 + r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)\left(r_{AB} + \mathrm{e}^{-\mathrm{i}\beta_A}\right)\left(1 - \mathrm{e}^{-2\mathrm{i}\beta_B}\right)\right\}, & (37)\\
\Delta m_{21} &= \mathrm{i}\frac{\Delta N_B}{N_B} U_{AB}N_A\left\{\beta_B\left[\left(1 - r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)^2 + \mathrm{e}^{-2\mathrm{i}\beta_B}\left(r_{AB} - \mathrm{e}^{-\mathrm{i}\beta_A}\right)^2\right]\right.\\
&+ \left.\mathrm{i}\left(1 - r_{AB}\mathrm{e}^{-\mathrm{i}\beta_A}\right)\left(r_{AB} - \mathrm{e}^{-\mathrm{i}\beta_A}\right)\left(1 - \mathrm{e}^{-2\mathrm{i}\beta_B}\right)\right\}. & (38)
\end{aligned}$$

In these expressions we recognize the contributions originating from the propagation and interface effects proportional to β_B and $\left(1 - \mathrm{e}^{-2\mathrm{i}\beta_B}\right)$, respectively.

3.2 *Ultrathin film approximations*

When the central magnetic layer B is ultrathin we can put for the elements of the characteristic matrix in the symmetric unit

$$m_{11} \approx \cos\beta_A, \qquad m_{12} \approx \frac{\mathrm{i}}{N_A}\sin\beta_A, \qquad m_{21} \approx \mathrm{i}N_A\sin\beta_A. \tag{39}$$

and

$$\begin{aligned}
\Delta m_{11} &\approx -\beta_B \sin\beta_A \frac{\Delta N_B}{N_A}, & (40)\\
\Delta m_{12} &\approx -\mathrm{i}\beta_B(1 - \cos\beta_A)\frac{\Delta N_B}{N_A^2}, & (41)\\
\Delta m_{21} &\approx \mathrm{i}\beta_B(1 + \cos\beta_A)\Delta N_B. & (42)
\end{aligned}$$

In order to account for the optical effect of B layers we retain the argument of the Chebyshev polynomials in the form given in Eq. (32). Then the MO response in reflection of the periodic structure can be estimated from

$$\chi_r^{(0,qL,A)} \approx \frac{4N_0\beta_B\Delta N_B p_q}{\left(N_0^2 - N_A^2\right)\left(p_q\mathrm{e}^{\mathrm{i}\beta_A} - p_{q-1}\right)}, \tag{43}$$

If the optical effect of B layers may be completely neglected the argument of the Chebyshev polynomials is replaced by $\cos\beta_A$. Then, making use of the properties of Chebyshev polynomials

$$\begin{aligned} \mathrm{e}^{\mathrm{i}q\beta_A} &= p_q \mathrm{e}^{\mathrm{i}\beta_A} - p_{q-1} \\ p_q &= \mathrm{e}^{\mathrm{i}(q-1)\beta_A}\left(1+\mathrm{e}^{-2\mathrm{i}\beta_A}+\mathrm{e}^{-4\mathrm{i}\beta_A}+\cdots+\mathrm{e}^{-2\mathrm{i}(q-1)\beta_A}\right), \end{aligned} \tag{44}$$

we arrive at a simplified representation

$$\chi_r^{(0,qL,A)} \approx \frac{4N_0\beta_B\Delta N_B \mathrm{e}^{-\mathrm{i}\beta_A}\left(1-\mathrm{e}^{-2\mathrm{i}q\beta_A}\right)}{\left(N_0^2-N_A^2\right)\left(1-\mathrm{e}^{-2\mathrm{i}\beta_A}\right)}. \tag{45}$$

The formula may be used for a qualitative evaluation of trends in $\chi_r^{(0,qL,A)}$ when the multilayer parameters change. It is consistent with the saturation of the effect in absorbing multilayers for a large number of periods. The ultrathin film approximation in its simplest form [11], ignores the optical effects of non–magnetic layers. This leads to an expression linear in the number of periods, q,

$$\chi_r^{(0,qL,2)} \approx \frac{4q\beta_B N_0 \Delta N_B}{N_0^2-N_2^2}. \tag{46}$$

4 Examples

In this section, the analytical formulae derived above will be evaluated numerically. The examples are selected from the category of cobalt containing multilayers combined with the transition (platinum, palladium, chromium) and noble metals (copper, gold, silver). These systems form the subject of recent research in MO materials and spin electronics.

We now apply the expressions for $\chi_r^{(0,qL,A)}$ to magnetic multilayers consisting of symmetric A/B/A blocks, A=Pt or Cu, B=Co. At a radiation wavelength of 632.8 nm, the Co, Pt and Cu layers will be characterized by $\varepsilon_{xx}^{(\mathrm{Co})} = -12.5036 - \mathrm{i}18.4639 = (2.21 - \mathrm{i}4.17)^2$ [16], $\varepsilon_{xy}^{(\mathrm{Co})} = -0.7410 + \mathrm{i}0.2077$, giving $\Delta N^{(\mathrm{Co})} = 0.0590 - \mathrm{i}0.0562$ [17], $N^{(\mathrm{Pt})} = 2.33 - \mathrm{i}4.14$ [18], and $N^{(\mathrm{Cu})} = 0.24 - \mathrm{i}3.42$ [19], respectively. The simulation results are displayed in Fig. 2. The figure compares the curves computed with Eq. (31), which is equivalent to the 4×4–matrix formalism, with the approximate formulae given by Eqs. (45) and (46). Figure 2 shows that the ultrathin approximation (46), applied for all magnetic and nonmagnetic films, differs significantly from the complete treatment. On the other hand, the approximation (45), considering the optical effects of nonmagnetic films, describes attenuation and saturation of the MO angles. The agreement between the complete treatment with Eq. (31) and approximate Eq.(45) is better for A=Pt, Pd or Cr than for noble metals A=Cu, Au or Ag. The latter group being characterized by the optical constants strongly different from those in transition metals. The approximations are slightly improved when Eq. (43) instead of Eq. (45) is employed.

We observe that the range of magnetic film thicknesses where the MO rotation and ellipticity amplitudes are satisfactorily reproduced is rather narrow. Note that in the MO magnetometry of magnetic multilayers the exact computation of the amplitudes is rarely of primary interest as they can be rarely checked by experiment. This is mostly due to the fact that the parameters of the ultrathin films, required as the input data in the computation, are often known with a limited accuracy. In addition, they strongly depend on the deposition process and vary with thickness. The approximate formulae still remain of interest as they provide an information on the trends in MO rotations and ellipticities. This allows one to choose the (laser) wavelengths where MO magnetometry

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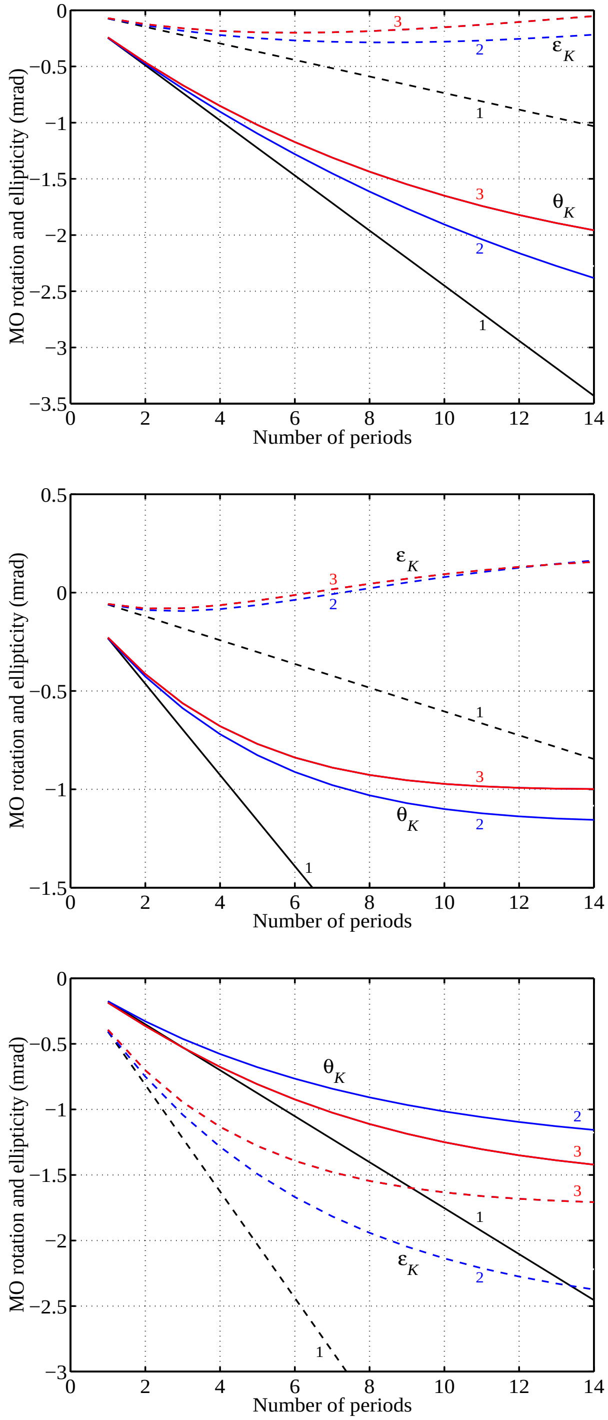

Pt	(0.4 nm)
Co	(0.4 nm)
Pt	(0.4 nm)
⋮	

Pt	(1.2 nm)
Co	(0.4 nm)
Pt	(1.2 nm)
⋮	

Cu	(1.2 nm)
Co	(0.4 nm)
Cu	(1.2 nm)
⋮	

Fig. 2. Magnetooptic azimuth rotation (full lines), θ_K, and ellipticity (dashed), ϵ_K, in a periodic multilayer consisting of symmetric A/B/A blocks, Pt(0.4 nm)/Co(0.4 nm)/Pt(0.4 nm), Pt(1.2 nm)/Co(0.4 nm)/Pt(1.2 nm), and Cu(1.2 nm)/Co(0.4 nm)/Cu(1.2 nm), as a function of number of the blocks. The curves 1 correspond to the ultrathin film approximation applied to all layers (Eq. (46)), the curves 2 were obtained with the ultrathin film approximation applied selectively to the magnetic layers using Eq. (45). The curves 3 were computed with Eq. (31).

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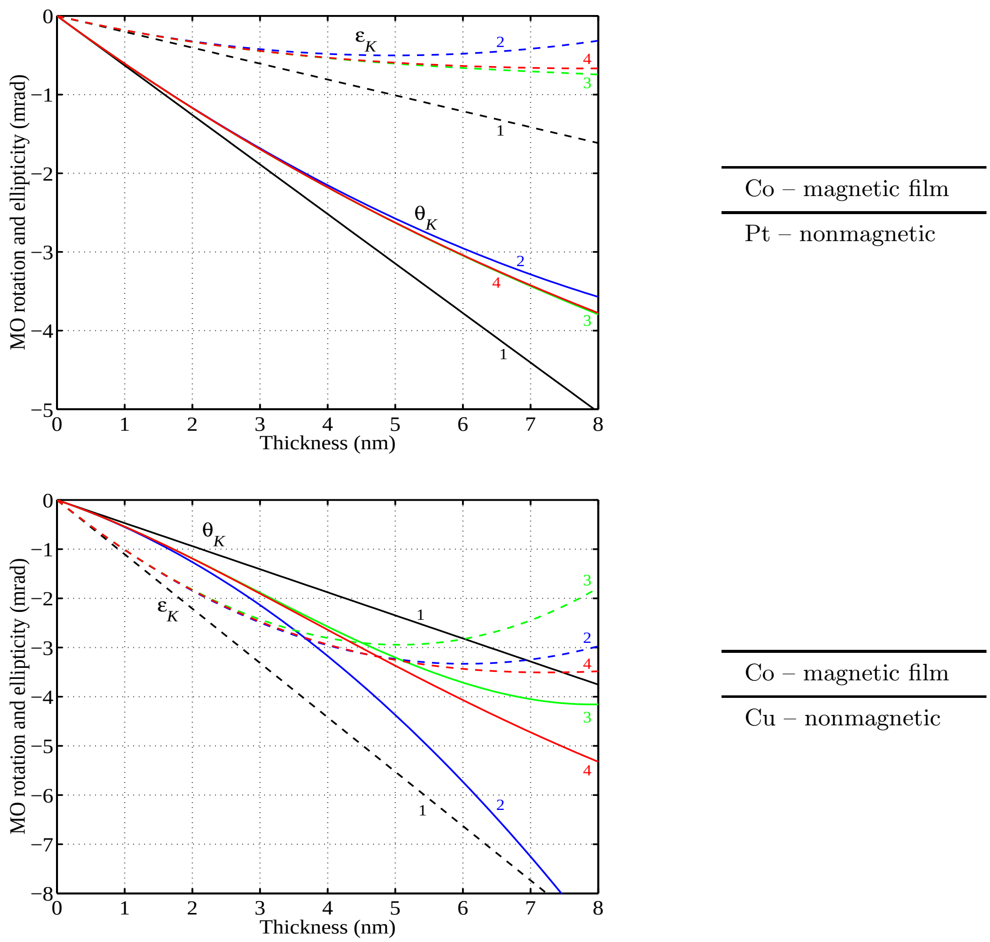

Fig. 3. The effect of the Co film thickness, $d^{(Co)}$, on the reflection MO azimuth rotation, θ_K (full lines), and ellipticity, ϵ_K, (dashed) at the wavelength $\lambda = 632.8$ nm. The Co film is deposited on Pt or Cu substrate. The curves 1, 2 and, 3 correspond to the approximations limited to first, second (Eq.(48)), and third orders in $\beta^{(\mathrm{Co})} = (2\pi/\lambda)N^{(\mathrm{Co})}d^{(Co)}$. The curves 4 were obtained without restriction on the magnetic layer thickness (Eq.(47)).

(e.g., hysteresis loop tracing) in a particular multilayer system under investigation can be performed with an optimum sensitivity.

To investigate the problem of ultrathin film approximation in more details, we consider a simple case of a magnetic layer (1) on a non–magnetic substrate (2). For the reflection MO response, assuming small azimuth rotations and ellipticities, we have

$$\begin{aligned}\chi_r^{(0,1,2)} &= \frac{\Delta N_1}{2N_1}\left(1+\mathrm{e}^{-2\mathrm{i}\beta_1}r_{01}r_{12}\right)^{-1}\left(r_{01}+\mathrm{e}^{-2\mathrm{i}\beta_1}r_{12}\right)^{-1} \\ &\times \left(1-r_{01}^2\right)\left[4\beta_1 r_{12}\mathrm{e}^{-2\mathrm{i}\beta_1} - \mathrm{i}\left(1-\mathrm{e}^{-2\mathrm{i}\beta_1}\right)\left(1+r_{12}^2\mathrm{e}^{-2\mathrm{i}\beta_1}\right)\right]. \qquad (47)\end{aligned}$$

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The formula provides results which are practically equivalent to those obtained with the 4×4–matrix formalism. Its development, up to second degree in β_1, gives

$$\chi_r^{(0,1,2)} \approx \frac{4(\Delta N_1)N_0}{N_0^2 - N_2^2}\left[\beta_1 + \frac{N_2}{N_1}\left(1 - 2\frac{N_0^2 - N_1^2}{N_0^2 - N_2^2}\right)\beta_1^2\right]. \tag{48}$$

The term linear in β_1 agrees with the previous result [20]. In Figure 3 the comparison of the exact calculation with the approximations is displayed for the case of a Co layer on a thick Pt or Cu substrate, respectively. The film thickness range where the approximation is valid depends on the choice of the substrate material as well as on the wavelength. As a rule, it differs for the rotation and ellipticity. We observe that in the case of Pt substrate the linear, quadratic and cubic approximations are meaningful for Co thicknesses below 1 nm, 3.5 nm, and 6 nm, respectively, while in the case of Cu substrate these ranges reduce to 0.5 nm, 1.5 nm, and 3.5 nm, respectively. This shows that the ultrathin film approximation restricted to the linear term in Eq. (48) as applied, for example, in Ref. [11], can reproduce the exact calculation only in a very limited range of thicknesses. An extension of this range may be achieved by the inclusion of the second and third order terms.

5 Conclusions

The transmission and reflection magnetooptic effects in periodic magnetic multilayers were described using the formalism originally developed for isotropic multilayers. The simplifications are made possible thanks to the restriction to the normal light incidence and polar magnetization. The formulae were provided for magnetic multilayers consisting of symmetrical blocks. They give results which are completely equivalent to those obtained by a universal 4×4–matrix formalism. Approximate expressions evaluating the effect of various multilayer parameters on the magnetooptic response, were obtained. Both exact and approximate formulae were evaluated numerically in Co/Pt and Co/Cu multilayers. The range of magnetic film thicknesses where the approximations well reproduce the results of the exact formulae is rather narrow and depends on the wavelength and polarization state of the incident radiation. Nevertheless the approximate formulae remain useful even beyond this range as they correctly predict the main trends in the MO effects. In particular, they may be helpful in the choice of the optimum radiation wavelenght for a particular multilayer system investigated by the MO magnetometry.

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Appendix: Chebyshev polynomials

A few first of them along with their derivatives are given by

$$\begin{array}{ll}
p_0 = 0, & p_6 = 32m_{11}^5 - 32m_{11}^3 + 6m_{11}, \\
p_1 = 1, & p_7 = 64m_{11}^6 - 80m_{11}^4 + 24m_{11}^2 - 1, \\
p_2 = 2m_{11}, & p_8 = 128m_{11}^7 - 192m_{11}^5 + 80m_{11}^3 - 8m_{11}, \\
p_3 = 4m_{11}^2 - 1, & p_9 = 256m_{11}^8 - 448m_{11}^6 + 240m_{11}^4 - 40m_{11}^2 + 1, \\
p_4 = 8m_{11}^3 - 4m_{11}, & p_{10} = 512m_{11}^9 - 1024m_{11}^7 + 672m_{11}^5 - \\
p_5 = 16m_{11}^4 - 12m_{11}^2 + 1, & \qquad -160m_{11}^3 + 10m_{11},
\end{array} \tag{49}$$

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$$
\begin{array}{llll}
[p_0]' = 0, & [p_4]' = 6p_3 + 2p_1, & [p_8]' = 14p_7 + 10p_5 + 6p_3 + 2p_1, & \\
[p_1]' = 0, & [p_5]' = 8p_4 + 4p_2, & [p_9]' = 16p_8 + 12p_6 + 8p_4 + 4p_2, & \\
[p_2]' = 2p_1 = 2, & [p_6]' = 10p_5 + 6p_3 + 2p_1, & [p_{10}]' = 18p_9 + 14p_7 + 10p_5 + & \\
[p_3]' = 4p_2, & [p_7]' = 12p_6 + 8p_4 + 4p_2, & \qquad +6p_3 + 2p_1. &
\end{array} \tag{50}
$$

Higher orders are obtained using the recursion relations ($q \geq 0$):

$$
\begin{aligned}
p_{q+1} &= 2m_{11}p_q - p_{q-1}, \qquad & p_{q+1}^2 - 2m_{11}p_q p_{q+1} + p_q^2 = 1, \\
p'_{q+2} &= 2(q+1)p_{q+1} + p'_q, \qquad & p_q^2 - p_{q+1}p_{q-1} - 1 = 0, \\
p'_{q+1} &= \frac{p'_q[m_{11}p_{q+1} - p_q] + p_q p_{q+1}}{p_{q+1} - m_{11}p_q}. &
\end{aligned} \tag{51}
$$